Journal of Engineering Physics and Thermophysics, Vol. 81, No. 6, 2008

HEAT AND MASS TRANSFER IN DISPERSE AND POROUS MEDIA

MATHEMATICAL DESCRIPTION OF MOTION AND ADHERENCE OF DROPLETS TO THE FLOW BOUNDARIES

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UDC 517.958:536.2

A mathematical model of the mechanics of heterogeneous media is suggested to describe the icing of the surfaces of bodies placed into an atmosphere containing supercooled droplets of liquid. Verification of the model by using the dependence of the temperature of a body in a flow on time has proved the adequacy of the model.

In using some of the technical facilities in the atmosphere that contains supercooled droplets of liquid, different undesirable phenomena occur, among which is precipitation of fine droplets onto the surface of these facilities with their subsequent freezing. In aviation this phenomenon has been known for long and is called icing. In the present work we will refer the term "icing" to the process of phase transition of a liquid into a solid state occurring on the boundaries of the bodies in a mobile or a motionless atmosphere containing fine droplets of a supercooled liquid that precipitate on the surfaces of the bodies.

As an example we may point to the phenomenon of the icing of airplanes moving, e.g., at subsonic velocities while taking off and landing. Moreover, the droplets present in a supercooled unstable state far from a flying vehicle may cause, while colliding, the process of spontaneous crystal formation. As a result of the development of this process, icing of possibly two types may form on the boundaries of a body: a frost or a transparent glass-like ice. This deteriorates the aerodynamic characteristics of a flying vehicle and causes loss of aircraft controllability and of its stability. Cases of icing where a glass-like ice grew catastrophically in minutes on the surfaces of an airplane have been reported.

We note that the presence of supercooled droplets in an air flow leads to dropwise icing. In the presence of water vapor that sublimes into ice particles sublimation icing may also be observed.

Some of the results of studies of the phenomenon of icing can be found in monographs [1–3] devoted to the introduction to the subject. They consider, in particular, the dynamic and thermodynamic problems of icing, cite mathematical models in the form of equations of motion of droplets in an air flow, and suggest a simplified boundary condition on the flying vehicle surface that describes the adherence of droplets. The notion of the coefficient of precipitation $E = \Delta y/\Delta s$ has been introduced as a quantity equal to the ratio of the space between the trajectories of the particles Δy initially far from the body to the space between these very particles Δs when they settled onto the body. Also equations to describe the thermodynamics of droplets are given; they express the heat balance of a discrete phase on the flying vehicle surface. Technical applications to the problems of protection of airplanes and helicopters from icing are described, as well as practical pieces of advice and recommendations on taking flights in icing conditions.

Useful information on the physics of freezing is given also in [4], including that pertaining to the problem of the icing of ships. In particular, the problems of heat exchange between the ice and the atmosphere, the problems of thermal expansion, the mechanical properties of ice, the influence of ice on the surrounding medium, etc, were considered. In order to carry out physicomathematical modeling of the dynamic phenomena of freezing, thermal models with free boundaries are used (the Stefan problem). Changes in the stressed-deformed state of ice masses are described by models of elastic and inelastic behavior of loaded solids. A large body of long-standing observational data on the

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deposition of an ice mantle are given. Apart from the possible icing of airplanes and ships, the literature also describes cases of the icing of high-rise buildings, aerial power lines, automobiles, trains, glazed roads, etc. As is seen, the problem is of current interest in many practical activities of human beings.

We will dwell on the problem of physical-mathematical description of the given phenomenon within the framework of the mechanics of heterogeneous media (MHM). Thereby we arrive at the statement and solution of the problems of interaction of heterogeneous flows with flow boundaries. For this purpose we will consider a certain solid body moving in a heterogeneous medium formed by a mixture of a gas with fine liquid droplets. The presence of supercooled droplets of micron size leads to dropwise icing of a body.

In order to describe a gas suspension flow of supercooled droplets around a body, we may apply the theory of interacting continua. However, it should be taken into consideration that under normal conditions the volume concentration of droplets in the atmosphere is very small and is estimated by the values of the order of 10^{-7}–10^{-8}. This means that droplets exert a weak influence on the gas parameters, and the problem of flow around a body can be considered at the first stage within the framework of the regime of single particles, i.e., within the Euler-Lagrange approach, with first the field of the gas flow around the body being calculated and then the fields of particle flows being determined. This approach is rather well known in the literature.

The mathematical model of icing in the regime of solitary supercooled droplets that consists of the equations of motion of a liquid droplet in the gas flow field and droplet energy conservation equation will be written in a dimensionless form as

$$\frac{d\mathbf{u}_2}{dt} = -\left[1 + \varphi\,(\mathrm{Re})\right] (\mathbf{u}_2 - \mathbf{u}_1)\,, \qquad \frac{d\mathbf{x}_2}{dt} = \mathbf{u}_2\,,$$

$$\frac{dT_2}{dt} = -\frac{T_2 - T_1}{3\mathrm{Pr}} \frac{2 + a\mathrm{Re}^n \mathrm{Pr}^{1/3}}{1 + 3.18\mathrm{Kn}}\,, \tag{1}$$

where $\mathbf{u}_i = (u_i,\ v_i)$ and T_i are the vectors of the velocity and temperature of the gas ($i = 1$) and of the droplet ($i = 2$); $\mathbf{x}_2$ is the vector of the position of the droplet in space; $\mathrm{Pr} = c_2\mu/\lambda$. The dimensionless quantities are determined with the aid of the characteristic time of the Stokesian relaxation of velocities $\tau_{\mathrm{St}} = 2\rho_{22}r_2^2/(9\mu)$, of the density ρ_{22} and radius r_2 of the droplet temperature T_0, and the velocity scale u_0. As the latter we have taken the gas velocity in the initial section of the flow located far from the body. The initial conditions for the system of equations (1) will be taken in the form of the Cauchy conditions:

$$t = 0: \quad \mathbf{u}_2 = \mathbf{x}_2 = 0\,, \quad T_2 = 1\,. \tag{2}$$

The expression $\varphi(\mathrm{Re})$ on the right-hand sides of Eqs. (1) is a correction to the Stokes resistance law. In the continuum approximation the resistance law has the form

$$1 + \varphi\,(\mathrm{Re}) = 1 + 0.15\mathrm{Re}^{0.687}\,, \tag{3}$$

and under the conditions of the rarefaction of the atmosphere the resistance law for droplets has the following correlation (the Henderson formula):

$$1 + \varphi\,(\mathrm{Re}) = \left(1 + 0.15\mathrm{Re}^{0.687}\right) \frac{1 + \exp\left(-\dfrac{0.427}{\mathrm{M}^{4.63}} - \dfrac{3.0}{\mathrm{Re}^{0.88}}\right)}{1 + \dfrac{\mathrm{M}}{\mathrm{Re}}\left[3.82 + 1.28\exp\left(-1.25\,\dfrac{\mathrm{Re}}{\mathrm{M}}\right)\right]}\,. \tag{4}$$

Here $\mathrm{M} = \left|\mathbf{u}_1 - \mathbf{u}_2\right|/c$, c is the dimensionless speed of sound;

TABLE 1. Characteristics of the Temperature Relaxation Zone for Various Velocities and Temperatures of a Cooled Gas

u_1, m/sec	T_1, °C	t_*, sec	Resistance law (3)		Resistance law (4)	
			L_T, m	T_*, °C	L_T, m	T_*, °C
10	–20	0.18	1.75	–17.08	1.75	–17.08
	–40	0.21	2.05	–37.06	2.04	–37.06
	–60	0.24	2.28	–57.00	2.26	–57.00
50	–20	0.15	7.49	–17.03	7.49	–17.04
	–40	0.18	8.98	–37.01	8.98	–37.02
	–60	0.21	10.17	–57.06	10.17	–57.07
100	–20	0.14	13.94	–17.03	13.93	–17.04
	–40	0.17	16.91	–37.01	16.91	–37.02
	–60	0.19	19.29	–57.06	19.29	–57.07

$$\mathrm{Re} = \mathrm{Re}_0\, symbol \mathbf{u}_1 - \mathbf{u}_2 symbol\,; \quad \mathrm{Re}_0 = 2\,\frac{\rho_{11} r_2 u_0}{\mu}\,; \qquad symbol \mathbf{u}_1 - \mathbf{u}_2 symbol = \left[(u_1 - u_2)^2 + (v_1 - v_2)^2 \right]^{1/2}\,;$$

ρ_{11} is the gas density. In the derivation of Eqs. (1)–(4) allowance was made for the correlation formula that describes heat transfer in a continuum regime:

$$\mathrm{Nu}_0 = 2 + a\mathrm{Re}^n \mathrm{Pr}^{1/3}\,,$$

where a and n are constants of the heat transfer law for a droplet.

As the size of droplets decreases, the heat transfer rate starts to depend on the rarefaction criterion, that is, the Knudsen number Kn. Heat transfer of very small droplets is described by the following correlation formula that describes experimental data on the heat transfer of spheres:

$$\mathrm{Nu} = \frac{\mathrm{Nu}_0}{1 + 3.18\mathrm{Kn}}\,, \quad 10^{-3} \le \mathrm{Kn} \le 10^2\,, \quad \mathrm{Kn} = \frac{\mathrm{M}}{\mathrm{Re}} \sqrt{\frac{\gamma\pi}{2}}\,,$$

where Re << 1. Thus, the problem of flow of a two-phase rarefied gas suspension around a body has been reduced to the solution of the Cauchy problem (1), (2) supplemented with the corresponding boundary conditions on the body. We will dwell on the example of solving the problem of motion and cooling of a droplet that was heated by a burner to room temperature in the prechamber of an aeroclimatic tube (the problem is also a test one).

Let us assume that droplets of a liquid under normal conditions are injected into a certain volume. The droplets find themselves in a negative-temperature gas flow that models the conditions of flight of a technical facility in the conditions of possible icing. It is required to determine the distance at which the velocity and temperature of the particles will reach equilibrium values, i.e., the values that characterize the velocity and temperature of the cooled gas. With this aim in view, calculations of the Cauchy problem (1), (2) were carried out for a droplet of radius r_2 = $4 \cdot 10^{-5}$ m at different values of the environment temperature T_1 and of the blowing flow velocity u_1. Table 1 contains the data obtained for the time of relaxation t_* of the droplet temperature, relaxation zone length L_T, and of the final droplet temperature T_* at the end of the relaxation zone.

Omitting a detailed analysis of the calculated data, we note that the length of the zone of temperature relaxation is equal to L_T = 2–19 m depending on the velocity of the cooled air flow and its temperature. This indicates that the length of the prechamber (the volume into which the droplets are injected) must not be less than L_T.

We will find the coefficient of precipitation E, and for this purpose we will consider a gas flow past a cylinder the velocity field of which is prescribed as follows

$$u_1 = u_0 \left[1 + \frac{R^2}{r^2} \left(1 - \frac{2x^2}{r^2} \right) \right], \quad v_1 = -\, u_0 xy\, \frac{2R^2}{r^4}\,,$$

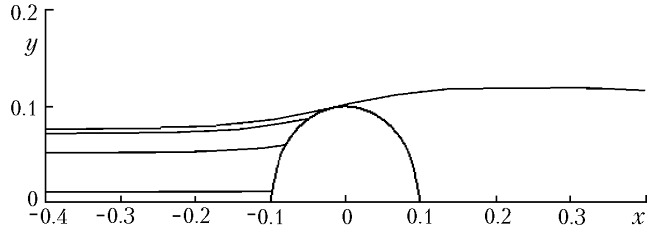

Fig. 1. Limiting trajectory of droplets that determines the coefficient of deposition on the cylinder.

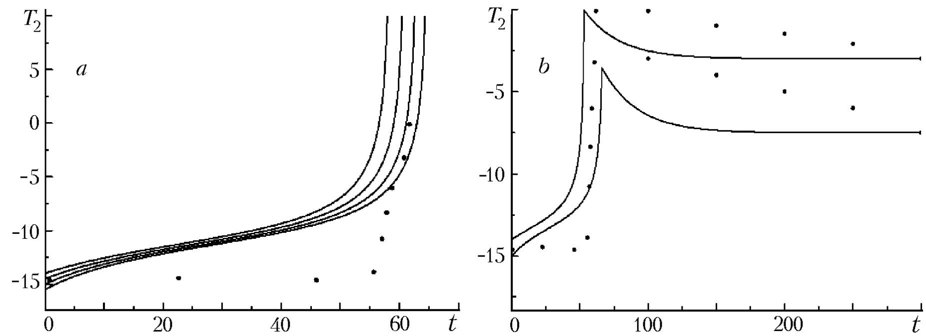

Fig. 2. Time distribution of temperature at different points on the cylinder surface (a) and same with allowance for cooling (b). $T_1 = -17.5^{o}$C; T_2, ^{o}C; t, sec.

where R is the cylinder radius. The limiting trajectory that determines the coefficient of precipitation on the cylinder is shown in Fig. 1. The calculations have shown that the precipitation coefficient E obtained by us is close to that given in [1] and equal to 0.7.

We will also consider the phenomenon of spontaneous crystallization of droplets on a profile. There are few works related to the physics proper of the crystallization of liquid droplets moving in a high-speed flow of a low-temperature gas when they fall out onto an aerofoil profile. Mention should be made here of work [3] that gives the results of the thermophysical investigations of this phenomenon, when a hollow cylinder is used as an investigated aerofoil profile. The temperature of the incoming two-phase flow was equal to $T_1 = 17.5^{o}$C, the velocity to $v = 78.5$ m/sec, and water content to 0.42 g/m^3, which corresponded to the volume concentration ~$4.2 \cdot 10^{-7}$, with the diameter of droplets varying from 39 to 45 μm. Interesting data are also given on the change in the temperature in time at different points on the cylinder surface. They explicitly point to the activation character of the phenomenon of crystallization. Indeed, for some time the temperature at the frontal point of the cylinder remains approximately constant [1]. We will estimate this stagnation temperature from the formula $T = T_0 + v^2/2c_2$ and obtain $T = -14.6^{o}$C (i.e., a value close to that obtained in the experiment). We will refer to the time during which the temperature on the cylinder surface remains approximately constant as the crystallization induction time.

The foregoing allows us to write the equation of the droplet phase energy conservation on the cylinder surface in the form

$$N \frac{4}{3} \pi r^3 \rho_{22} c_2 \frac{dT_2}{dt} = - N\alpha 4\pi r^2 (T_2 - T_1) + NL_3 4\pi r^2 \rho_{23} K (T_{\text{m}} - T_2) \exp\left(-\frac{E_a}{RT_2}\right). \quad (5)$$

Here L_3 is the latent (specific) heat of ice formation; $\rho_{22} = 1020$ kg/m^3 is the density of a droplet; $\rho_{23} = 920$ kg/m^3 is the ice density; $T_{\text{m}} = 273$ K; $T_1 = 253$ K is the aerofoil profile temperature; $c_2 = 4.1868 \cdot 10^3$ J/(kg·K) is the water heat capacity; N the quantity of droplets falling out on the profile surface; K the pre-exponential factor, and α is the heat transfer coefficient. It has been shown that this mathematical model allows one to carry out calculations with the physically acceptable activation energy (E_{a}) values. Some of the results of calculation by model (5) are presented in Fig. 2a. It presents calculated temperature-vs.-time curves for different points on the cylinder profile, with the data of the experiment described in [3] being represented by dots. As we can see, the time of retardation of icing is described rather satisfactorily within the framework of the given model. We note that we carried out the calculation up to the moment of the catastrophic rise in the temperature. As soon as an infinite gradient had appeared on the temperature profile, the calculation was terminated and thereafter the heat release term was detached from the crystallization. This was followed by cooling of the formed ice crust during the time of cooling relaxation, as shown in Fig. 2b. We see that the coincidence of the predicted and experimental data can be considered satisfactory.

Thus, in order to describe the process of icing of an aerofoil profile in a flow of a mixture of a gas and supercooled droplets within the framework of the mechanics of heterogeneous two-velocity two-temperature media, a mathematical model of the regime of single supercooled droplets has been suggested. The model has been tested using the dependences of the temperature relaxation zone length on the initial temperature of the gas and its velocity, data on the adhesion coefficient, and the experimental data of ITAM on the dependence of temperature at the frontal point of a cylinder in a flow of supercooled droplets.

This work was carried out within the framework of the intertopical integration project "Investigation of the Physical Processes That Cause Icing of Flying Vehicle" of the Siberian Branch of the Russian Academy of Sciences.

NOTATION

c_2, heat capacity of a droplet; E, coefficient of precipitation; E_a, activation energy; Kn, Knudsen number; M, Mach number; Pr, Prandtl number; r_2, radius of a droplet; Re, Reynolds number; t, time; T_i, temperature of a gas (i = 1) and droplet (i = 2); $\mathbf{u}_i$, vectors of the velocity of a gas (i = 1) and droplet (i = 2); x, y, dimensionless Cartesian coordinates; λ, thermal conductivity coefficient; μ, gas viscosity; ρ_{11}, gas density; ρ_{22}, droplet density. Subscripts: a, activation; m, melting.

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